HOW HIGH IS THE BUILDING?

You are sitting in a room on an unknown floor of a high-rise looking out a 1.5 meters tall window. Someone drops a ball from the roof top. It takes the ball .8 seconds to pass from the top to the bottom of your window. The ball falls to the ground, bounces elastically (i.e., no energy lost) and reappears at your window 12 seconds later. How high is the building.

1. Draw a sketch and coordinate axis, then put as much information as you can in on the sketch.

2.) This problem is obscure enough and complex enough to be a real headache to start. When something like that happens, the best thing to do is to go to the part of the problem you know the most about. In this case, that would be the section of freefall associated with the window. You know how far the object falls as it passes the window (1.5 m) and how long it takes to go that distance (.2 seconds). With that information, you use

$$\Delta y_w = v_2 \Delta t_w + \frac{1}{2} a_y \left(\Delta t_w \right)^2$$

to figure out how fast it is moving just as it gets to the window top.

3.) To determine how fast it's moving at the window's bottom, use

$$v_1 = v_2 + a_y \Delta t_w$$

4.) With the time to bottom of window (half of 12 seconds) and the window-bottom velocities, you can get the distance from ground to window bottom using

$$\Delta y_b = v_1 \Delta t_b + \left(\frac{1}{2} \right) a_y \left(\Delta t_b \right)^2$$

5.) And with the window top velocity and the fact that the freefall started from rest (i.e., v=0), you can get the distance from the window top to the top of the building using

$$v_2{}^2 = v_{top}{}^2 + 2a\Delta y_t$$

6.) Once you have all the distances, add them up for the total height.

So how high is the building? For an object free falling from the roof, we can write:

y_{top}

$v_{top} = 0$

$$v_2^2 = v_{top}^2 + 2a\Delta y_t$$

$$\Rightarrow \Delta y_t = \frac{v_2^2 - v_{top}^2}{2a}$$

Δt_t

$$\Rightarrow \Delta y_t = \frac{(6.52 m/s)^2 - (0)^2}{2(-9.8 m/s^2)}$$

Δy_t

$$\Rightarrow \Delta y_t = 2.17 m$$

$y_2 = y_1 + 1.5$

v_2

$$\Delta y_w = v_2 \Delta t_w + \frac{1}{2} a_y (\Delta t_w)^2$$

$$\Rightarrow (1.5 \text{ m}) = v_2(.2 \text{ s}) + \frac{1}{2}(-9.8 m/s^2)(.2 \text{ s})^2$$

window

$$\Rightarrow v_2 = -6.52 \text{ m/s}$$

$\Delta t_w = .2 \text{ sec}$

$\Delta y_w = .15 \text{ m}$

$$v_1 = v_2 + a_y \Delta t_w$$

$$\Rightarrow v_1 = -6.52 \text{ m/s} + (-9.8 \text{ m/s}^2)(.2 \text{ s})$$

y_1

$$\Rightarrow v_1 = -8.48 \text{ m/s}$$

v_1

$$\Delta y_b = v_1 \Delta t_b + \left(\frac{1}{2}\right) a_y (\Delta t_b)^2$$

$$\Rightarrow \Delta y_b = (-8.48 \text{ m/s})(6 \text{ sec}) + .5(-9.8 \text{ m/s})(6 \text{ sec})^2$$

$\Delta t_b = 6 \text{ sec}$

$$\Rightarrow = 227 \text{ m}$$

Δy_b

So

$$y_{top} = \Delta y_b + \Delta y_w + \Delta y_t$$

$$= (227 \text{ m}) + (1.5 \text{ m}) + (2.17 \text{ m})$$

$$= 230.67 \text{ m}$$

$y_{bot} = 0$